



2<sup>nd</sup> FINAL MATHEMATICAL CUP  
FMC 2020

September 30, 2020

SENIOR CATEGORY  
MARKING SCHEME

PLEASE NOTE THAT THIS IS JUST A MARKING SCHEME FOR GRADING OF THE OFFICIAL SOLUTION. OTHER IDEAS LEADING TO SOLUTION ARE GRADED ACCORDINGLY.

**Problem 1.**

For (1)	2 points
For (2)	2 points
For (3)	2 points
Proving contradiction that $a_1$ and $a_2$ being different	2 points
Finding the function	2 points

**Problem 2.**

For (1)	3 points
For (2)	3 points
For (3)	3 points
Finishing the proof	1 point

**Remark.** Construction of the point X worth 1 point and this point is not additive. Obtaining partial results, but not obtaining completely (1), (2) and (3) is awarded with zero points. Other solutions leading to solution are graded accordingly.

**Problem 3.**

a)	
Finding the last number on the paper	1 point
For $k = \frac{1}{3} \left( \sum (\text{new number})^3 - \sum (\text{initial number})^3 \right)$	3 points
Finishing the proof for a)	3 points
b)	
Calculating $k$	3 points

**Problem 4.**

For claiming and proving that: For a prime number $p$ such that $p \mid n$ , it holds $n < p^2$ .	3 points
For $p_1 p_2 \cdots p_k \leq n < p_{k+1}^2$	2 points
For both cases $k = 0$ and $k = 1$	1 point
Proving that $k \leq 4$	2 points
For case $k = 2$ , $k = 3$ and $k = 4$	2 points

**Remark.** Only exploring isolated cases, for example  $n=8$ , is worth 0 points.