



2nd FINAL MATHEMATICAL CUP
FMC 2020

September 30, 2020

JUNIOR CATEGORY
MARKING SCHEME

PLEASE NOTE THAT THIS IS JUST A MARKING SCHEME FOR GRADING OF THE OFFICIAL SOLUTION. OTHER IDEAS LEADING TO SOLUTION ARE GRADED ACCORDINGLY.

Problem 1.

Assuming contradiction and $d^2n^2 + d^3 = m^2$	2 points
Using condition and $2n^2 = d \cdot k$	1 point
Substitution and $m^2 = d^2n^2 + d^3 = \frac{4n^6}{k^2} + \frac{8n^6}{k^3}$	2 points
Obtaining $(mk^2)^2 = 4n^6(k^2 + 2k)$	3point
Finishing the proof	2 points

Problem 2.

For $(1+abc)^2(ab+bc+ca+a+b+c) \geq 3abc(ab+bc+ca+a+b+c+abc+1)$	2 point
Substitution for m, n, x	3 points
For $(m+n)(x^6 - x^3 + 1) \geq 3x^3(x^3 + 1)$	1 point
Using AM-GM	2 points
Finishing the proof	2 points

Problem 3.

Considering 2×2 square	3 points
Concluding that, $n \geq 2 \cdot (2k - 1) + 1 = 4k - 1$.	2 points
Proof for case $n = 4k - 1$	2 points
Construction for case $n = 4k$ and drawing one example	3 points

Remark: If the student has just drawing for $k \geq 2$ and nothing else, it is worth 1 point. This point is not additive with the other points. It can be given if the student has just the drawing.

Problem 4.

For (1)	3 points
For (2)	3 points
For (3)	3 points
Finishing the proof	1 point

Remark. Construction of the point X worth 1 point and this point is not additive.

Obtaining partial results, but not obtaining completely (1), (2) and (3) is awarded with zero points.
Other solutions leading to solution are graded accordingly.