Maths Beyond Limits

Mathematical programme for gifted high school students from all around the world!

The camp aims to create space for **development of young maths enthusiasts** through working on interesting and demanding subjects. It is designed to encourage participants to **share their knowledge** and passion with others as well as to enhance cooperation and integration of different mathematical societies.

MBL 2023 will take place on 11-24.09 in Poland

Typical day at MBL:

mathematical classes run in 80-minute-long sessions; they are devoted to some of the most beautiful concepts in mathematics from outside the high school's curriculum.

camper talks 30-minute-long presentations given by participants on topics connected with mathematics.

time academic unscheduled During this time all tutors and semitutors make themselves available for 'office hours' and groups often form to work together on problem sets.

evening activities non-mathematical activities run by participants or organisers in the evenings. They cover a wide range of workshops and give opportunities to try something new or improve soft skills.

Learn more!

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Applications are open until May 6th!

Maths Beyond Limits 2023 Qualifying Quiz

1. Kuba's favourite number n has the following properties:

- sum of its digits is equal to the product of its digits,
- exactly half of its digits are ones,
- n is the biggest number with these two properties.

Determine n.

2. Krzysztof, continuing his career in forestry, got a tree – a connected graph with n vertices and no cycles. He turned each edge into an arrow, making it a directed graph, but he disliked the effect. Now he can choose any vertex v such that all edges incident to v are going in it, and he can reverse them, so that they go out of v. Prove that after some number of moves it is possible for him to obtain any orientation of edges he likes.

3. Let n be a positive integer. Zana and Marianna are playing a game. At the beginning, they have the set $S = \{1, 2, \ldots, n\}$. They take turns alternately. In one turn they choose an element $k \in S$, and then they remove all divisors of k from S. The player who can't make a move, loses. Zana plays the first move. Who has the winning strategy?

4. Let ABC be an acute triangle. H is its orthocenter, and O is its circumcenter. Let P be the midpoint of BH and Q be the midpoint of CH. Prove that the circumcircles of triangles PHQ and ABC are tangent if and only if $OH||BC$.

5. Let S be a union of polygons sitting inside three dimensional space. We know that the projection of S onto each of the three planes $x = 0$, $y = 0$ and $z = 0$ is a square with side length 1. What is the minimal possible area of S?

6. We call a positive integer n *lucky*, if we can fill a unit square with n rectangles (without overlapping), such that each of them has side ratio equal to $2 : 3$.

a) Determine all lucky numbers.

We call a positive integer n *serendipitous*, if we can fill a cube with n rectangular parallelepipeds (without overlapping), each of them with side ratio $1 : 2 : 3$.

b) Prove that each integer $n \geqslant 100$ is serendipitous.

c) What is the smallest possible C such that all integers $n \geqslant C$ are serendipitous? (Finding the exact answer here might be hard, so we expect some bounds like ”C is greater than 15, but smaller than 73”. The points will be dependent on how good is your answer in comparison to answers of other applicants.)

7. Let $d(X,Y)$ be the distance between two points in a plane. A function $f : \mathbb{R}^2 \to \mathbb{R}^2$ satisfies $d(X,Y) = 1 \Rightarrow d(f(X), f(Y)) = 1$ for any points X, Y (in other words, if points X and Y are in distance 1 from each other, so are $f(X)$ and $f(Y)$).

a) Suppose f is injective, i.e. $f(X) \neq f(Y)$ if points X, Y are distinct. Does it imply that f is an isometry, i.e. $d(X,Y) = d(f(X), f(Y))$ for any X, Y?

b) What can you say when we don't require f to be injective?

8.

a) Let b_n be the number of labelled trees on n vertices (i.e. connected graphs without cycles, such that vertices are numbered from 1 to n). Prove that

$$b_n = \sum_{k=1}^{n-1} (n-k)\binom{n-2}{k-1} b_k b_{n-k}.$$

b) There are n witches sitting in a circle. Each witch has a hat with a number from 1 to n written on it (every number appears exactly once). In i-th minute (for $i = 1, 2, \ldots, n-1$) two witches swap places with each other. After $n-1$ minutes it turned out that for all $m = 1, 2, \ldots, n-1$ the witch with a number m is sitting on the place occupied at the start by the witch with a number $m+1$.

Let a_n be the number of possibilities in which this could happen. Prove that

$$a_n = \sum_{k=1}^{n-1} (n-k)\binom{n-2}{k-1} a_k a_{n-k}.$$

c) Deduce that $a_n = b_n$.

If you find any other interesting recurrence relations for sequences a_n and b_n, share them with us!